Intro to Proofs Day 8 Outline (This class meets for 110 minutes.)

**Section 3.2 worksheet, Quizzes, River crossing/bridge and torch problem, new proof portfolio**

***Remind about upcoming events on slides.***

PART 1: Synthesis Stuff (0-20minutes)

**Section 3.1 – Last Activity:**

* Demonstrate how I would look for a counterexample

**Synthesis Presentation** [0-20 minutes]

* Section 3.1 Exercise 1(a) on page 96
  + Prove that for all integers a, b, and c with a\neq 0, if a divides b and a divides c then a divides b-c.
  + There exist integers m and n such that am=b and an=c. So b-c = am-an = a(m-n). Then m-n is an integer so a divides b-c.
* Section 3.1 Exercise 9(a) on page 97
  + If a is congruent to 7 (mod 8) and b is congruent to 3 (mod 8) then a+b is congruent to 2 (mod 8)
  + We know 8 divides a-7 and 8 divides b-3. Then there exist integers m and n such that 8m = a-7 and 8n = b-3. So 8m+7 = a and 8n+3 = b. So a + b = 8m+8n+10 = 8(m+n+1) + 2. So 8 divides (a+b)-2 so a+b is congruent to 2 mod 8
* Section 3.1 Exercise 19(a) on pages 100-101
  + The proof is sort of correct, but definitely not well written. Need to say that there exists an integer n such that m=2n. Then say that 5n+2 is an integer by closure.
* Section 3.1 Exercise 19© on pages 100-101
  + This proof is incorrect and the statement is false.
  + False because 6 divides 9\*4 = 36, but 6 does not divide 9 and 6 does not divide 4. 6\*6 = 36, 6 = 3\*2 so 9\*4 = 3\*2\*6
  + In the proof, we can’t factor k=mn necessarily without one of those factors being 1. Also just because bc=mna does not mean that b=ma or c=na

PART 2: Section 3.2 Worksheet (20-55 minutes)

**Discuss Preview Activity [20-25]**

* Preview Activity from last week: Prove “if n^2 is odd then n is odd” vs. “if n is even then n^2 is even”:
  + If n^2 = 2k+1 then…. What? N = sqrt{2k+1}, not helpful.
* Slide 2: Assume in a direct proof of “for each integer n if 7 divides n^2-4 then 7 divides n-2”.
  + ASSUME that n is an integer. NOT “For each integer n…”
* Slide 3: Proof by contrapositive video questions
  + Sorry about the wrong video
* Slide 4: Assume in a proof by contrapositive of the same statement.
  + Same comment: ASSUME that n is an integer still. Does not become exists!
* Slide 5: Proof of biconditional video questions
* Slide 6: Write “for each integer a, 3 does not divide a if and only if 3 does not divide 2a^2 + 1”.
  + If and only if is the biconditional.
  + Leave the “for each integer a” part in BOTH statements.

**Section 3.2 Activity 1 [25-35]**

* Have students share examples of statements that they have written logically equivalent versions of.

**Section 3.2 Activity 2 [35-55]**

* Write down 2 statements to prove
* Students work at board with partner on conjecture

---------------------------------------------- BREAK ----------------------------------------------------------

PART 3: Existence proofs (65-95 minutes)

* Torch and bridge problem (3 groups)
* River crossing problem (2 groups)

PART 4: Quiz [95-110]

* Skills L1-L4

Preview Activity for next time: Proof by contradiction. Practice negating statements.

Synthesis activity for next time: Section 3.2 #5